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COMPARISON OF SIMPLE ACCELERATION METHOD AND CONWAY'S METHOD

Li Li Than*

Abstract

Congruence can be used to determine on which day of the week a given date falls. We discuss the calendar formula to calculate the day of the week of December 25, 2017. Also, we express easier formula to calculate. And then, we describe Conway's Doomsday Algorithm. The Doomsday Algorithm is an Algorithm for calculating the day of the week for any given calendar date. The algorithm is based on first computing doomsday which is the day of the week of the last day of February, or of January. We present an acceleration method for calculating the dooms year term of the Doomsday algorithm. Finally, we show that simple acceleration method is similar in form to Conway's lookup table acceleration method.

1. Definitionsm

A year is the amount of time it takes the Earth to make one complete orbit around the Sun.

A **day** is the amount of time it takes the Earth to make a complete rotation about the axis through its north and south poles.

A year is approximately 365.2422 days long. In 46 B.C., Julius Caesar (and its scientific advisors) compensated for this by creating the **Julian calendar**, containing a **leap year** every 4 years; that is, every fourth year has an extra day, namely, February 29, and so it contains 366 days. A **common year** is a year that is not a leap year.

This would be fine if the year were exactly 365.25 days long, but it has the fact of making the year $365.25 - 364.2422 = .0078$ (about 11 minutes 14 seconds) days too long. After 128 years, a full day was added to the calendar, that is, the Julian calendar over counted the number of days.

Let us now seek a calendar formula. For easier calculation, we choose 0000 as our reference year, even though there was no year! Assign a number to each day of the week, according to the following scheme:

Sun	Mon	Tues	Wed	Thurs	Fri	Sat
0	1	2	3	4	5	6

*. Dr, Lecturer, Department of Mathematics, Myingyan Degree Collage

In particular, March 1, 0000, has some number a , where $0 \le a \le 6$. In the next year, 0001, March 1 has number a +1(mod 7), for 365 days have elapsed from March 1, 0000, to March 1, 0001, and

$$365 = 52\times 7 + 1 \equiv 1 \bmod 7.$$

Similarly, March 1, 0002, has number a + 2, and March 1, 0003, has number a +3. However, March 1, 0004, has number a +5 , for February 29, 0004, fell between March 1, 0003, and March 1, 0004, and so $366 \equiv 2 \bmod 7$ days had elapsed since the previous March 1. We see, therefore, that every common year adds 1 to the previous number for March 1, while each leap year adds 2. Thus, if March 1, 0000, has number a, then the number a′ of March 1, year y, is

$$a' \equiv a + y + L \bmod 7,$$

[For 0000, $365 \equiv a \bmod 7$,
for 0001, $2 \times 365 \equiv a + 1 \bmod 7$,
for 0002, $3 \times 365 \equiv a + 2 \bmod 7$,
for 0003, $4 \times 365 \equiv a + 3 \bmod 7$,
for 0004, $4 \times 365 \equiv a + 4 + 1 \bmod 7$]

where L is the number of leap years from year 0000 to year y. To compute L, count all those years divisible by 4, then throw away all the century years, and then put back those back century years that are leap years. Thus,

$$L \equiv \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor,$$

where $\lfloor x \rfloor$ denotes the greatest integer in x.

For 1 year, L = 0,
for 2 years, L = 0,
for 3 years, L = 0,
for 4 years, L = 1,
and so on for 100 years , L = 24 = 100/ 4 -100/100,
for 400 years, L = 97 = 400/4 -400/100+400/400.

Therefore, we have

$$\begin{aligned} a' &\equiv a + y + L \\ &\equiv a + y + \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor \bmod 7. \end{aligned}$$

We can actually find a' by looking at a calendar. Since March 2, 1994, fell on a Tuesday,

$$2 \equiv a + 1994 + \lfloor 1994/4 \rfloor - \lfloor 1994/100 \rfloor + \lfloor 1994/400 \rfloor$$
$$\equiv a + 1994 + 498 - 19 + 4 \text{ mod } 7,$$

and so

$$a \equiv -2475 \equiv -4 \equiv 3 \text{mod} 7$$

(that is, March 1, year 0000, fell on Wednesday). One can now determine the day of the week on which March 1 will fall in any year $y > 0$, namely, the day corresponding to

$$3 + y + \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor \text{ mod } 7.$$

There is a reason we have been discussing March 1.

Let us now analyze February 28. For example, suppose that February 28, 1600, has number b. As 1600 is a leap year, February 29, 1600, occurs between February 28,1600, and February 28, 1601; hence 366 days have elapsed between these two February 28's, so that February 28, 1601, has number b+2. February 28, 1602, has number b+3, February 28, 1603, has number b+4, February 28, 1604, has number b+5, but February 28, 1605, has number b+7 (for there was a February 29 in 1604).

Let us compare the pattern of behavior of February 28, 1600, namely,

b, b+2, b+3, b+4, b+5, b+7, ..., with that of some date in 1599. If May 26, 1599, has the number c, then May 26, 1600, has the number c+2, for February 29, 1600, comes between these two May 26's, and so there are $366 \equiv 2 \text{mod} 7$ intervening days. The numbers of the next few May 26's, beginning with May 26, 1601, are c, c + 2, c + 3, c + 4, c + 5, c +7. We see that the pattern of the days for February 28, starting in 1600, is exactly the same as the pattern of the days for May 26, starting in 1599; indeed, the same is true for any date in January or February. Thus, the pattern of the days for any date in January or February of a year y is the same as the pattern for a date occurring in the preceding year $y - 1$: a year preceding a leap year adds 2 to the number for such a date, whereas all other years add 1. Therefore, February 28, 1600, has number b,

February 28, 1601, has number b + 2, (since 1600 is a leap year)

February 28, 1602, has number b + 3,

February 28, 1603, has number b + 4,

February 28, 1604, has number b + 5,

February 28, 1605, has number b+ 7, (since 1604 is a leap year)

So, it has the pattern: b, b+2, b+3, b+4, b+5, b+7,...,

May 26, 1599, has the number c,

May 26, 1600, has the number c + 2, (for February 29, 1600)

May 26, 1601, has the number c + 3,

May 26, 1602, has the number c + 4,

May 26, 1603, has the number c + 5,

May 26, 1604, has the number c + 7,

so it has the pattern:

$$c, c + 2, c + 3, c + 4, c + 5, c + 7.$$

Now we find the day corresponding to a date other than March 1. Since March 1,0000, has number 3, April 1, 0000, has number 6, for March has 31 days and $3+31\equiv 6 \bmod 7$. Since April has 30 days, May 1, 0000, has number $6+30 \equiv 1 \bmod 7$. Thus, the following table gives the number of the first day of each month in year 0000:

March 1,0000, has number	3
April 1	6
May 1	1
June 1	4
July 1	6
August 1	2
September 1	5
October 1	0
November 1	3
December 1	5
January 1	1
February 1	4

We are pretending that March is month 1, April month 2, etc. Let us denote these numbers by $1+j(m)$, where j(m), for m=1,2, ...,12, is defined by

j(m) : 2,5,0,3,5,1,4,6,2,4,0,3.

	day		j(m)		year	
For March 1, 0000 has number	1	+	2	+	0	$=3$
For March 1, 0001 has number	1	+	2	+	1	$=4$
For March 1, 0002 has number	1	+	2	+	2	$=5$
For March 1, 0003 has number	1	+	2	+	3	$=6$
For March 1, 0004 has number	1	+	2	+	4	$+1=8\equiv 1 \mod 7$
	1	+	2	+	$4+\lfloor 4/4\rfloor-\lfloor 4/100\rfloor+\lfloor 4/400\rfloor\equiv 1 \mod 7$	

⋮

For month m, day d, and year y, $d+j(m)+g(y) \mod 7$

where $g(y)=y+\lfloor y/4\rfloor-\lfloor y/100\rfloor+\lfloor y/400\rfloor$.

2. Calendar Formula and Its Application

Proposition 2.1. (Calendar Formula). The date with month m, day d, year y has number

$$d + j(m) + g(y) \mod 7 ,$$

where

$$j(m) = 2,5,0,3,5,1,4,6,2,4,0,3,$$

(March corresponds to m =1, April to m = 2, , February to m =12) and

$$g(y) = y + \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor,$$

provided that dates in January and February are treated as having occurred in the previous year.

Proof. The number mod 7 corresponding to month m, day 1, year y, is 1+ j(m) +g(y). It follows that 2+ j(m) +g(y) corresponds to moth m, day 2, year y, and, more generally, that d + j(m) +g(y) corresponds to month m, day d, year y.

Example 2.2. Let us use the calendar formula to find the day of the week on which December 25, 2017, fell. Here m= 4,d = 25,and y = 2017. Substituting in the formula, we obtain the number

$$25+ 4+ 2017+\lfloor 2017/ 4\rfloor - \lfloor 2017/100\rfloor + \lfloor 2017/ 400\rfloor = 2535 \equiv 1 \bmod 7,$$

therefore, December 25, 2017, fell on a Monday.

Most of us need paper and pencil (or calculator) to use the calendar formula in the proposition. Now we use some ways to calculation.

One mnemonic for j(m) is given by

$$j(m) = \lfloor 2.6m - 0.2 \rfloor, \text{ where } 1 \le m \le 12.$$

In above example, we also obtain the number

$$1+\lfloor (2.6)10-0\cdot 2\rfloor+2017+\lfloor 2017/4\rfloor-\lfloor 2017/100\rfloor+\lfloor 2017/400\rfloor = 2535 \equiv 1 \bmod 7$$

where m =10.

Another mnemonic for j(m) is in the sentence:

My	Uncle	Charles	has	eaten	a	cold	supper;	he	eats	nothing	hot.
2	5	$(7 \equiv 0)$	3	5	1	4	6	2	4	$(7 \equiv 0)$	3

2.3 Corollary. The date with month m, day d, year $y = 100C + N$, where $0 \le N \le 99$, has the number

$$d + j(m) + N + \lfloor N/4 \rfloor + \lfloor C/4 \rfloor - 2C \bmod 7,$$

provided that dates in January and February are treated as having occurred in the previous year.

Proof. If we write a year $y = 100C + N$, where $0 \le N \le 99$, then

$$y = 100C + N \equiv 2C + N \bmod 7,$$

$$\lfloor y/4 \rfloor = 25C + \lfloor N/4 \rfloor \equiv 4C + \lfloor N/4 \rfloor \bmod 7,$$

$$\lfloor y/100 \rfloor = C, \text{ and } \lfloor y/400 \rfloor = \lfloor C/4 \rfloor.$$

Therefore,

$$y + \lfloor y/4 \rfloor - \lfloor y/100 \rfloor + \lfloor y/400 \rfloor \equiv 2C + N + 4C + \lfloor N/4 \rfloor - C + \lfloor C/4 \rfloor \bmod 7$$
$$\equiv N + \lfloor N/4 \rfloor + \lfloor C/4 \rfloor - 2C \bmod 7.$$

This formula is simpler than the first one. For example, the number corresponding to December 25, 2017, is now obtained as

$$25 + 4 + 17 + \lfloor 17/4 \rfloor + \lfloor 20/4 \rfloor - 2(20) = 15 \equiv 1 \bmod 7.$$

Now I find the day of my birthday.

2.4. **Example** My birthday date is June 27, 1973. On what day of the week was I born?

If A is the number of the day, then

$$A \equiv 27 + 3 + 73 + \lfloor 73/4 \rfloor + \lfloor 19/4 \rfloor - 2(19) = 87 \equiv 3 \bmod 7.$$

I was born on a Wednesday.

3. Conway's Calendar Formula

John Horton Conway has found an even simpler calendar formula. In his system, he calls doomsday of a year that day of the week on which the last day of February occurs. For example, doomsday 1900, corresponding to February 28, 1900(1900 is not a leap year), is Wednesday = 3, while doomsday 2000, corresponding to February 29, 2000, is Tuesday = 2, as we know from Corollary 2.3.

Knowing the doomsday of a century year 100C, we can find the doomsday of any other year y = 100C + N in that century, as follows. Since 100C is a century year, the number of leap years from 100C to y does not involve the Gregorian alteration. Thus, if D is doomsday 100C (of course, $0 \le D \le 6$), then doomsday 100C + N is congruent to

$$D + N + \lfloor N/4 \rfloor \bmod 7.$$

For example, since doomsday 1900 is Wednesday = 3, we see that doomsday 1994 is Monday =1, for

$$3 + 94 + 23 = 120 \equiv 1 \bmod 7.$$

3.1 Proposition. (Conway's Formula). Let D be doomsday 100C, and let $0 \le N \le 99$. If $N = 12q + r$, where $0 \le r < 12$, then the formula for doomsday 100C + N is

$$D + q + r + \lfloor r/4 \rfloor \bmod 7.$$

Proof.

$$\begin{aligned} \text{Doomsday } (100C + N) &\equiv D + N + \lfloor N/4 \rfloor \\ &\equiv D + 12q + r + \lfloor (12q + r)/4 \rfloor \\ &\equiv D + 15q + r + \lfloor r/4 \rfloor \\ &\equiv D + q + r + \lfloor r/4 \rfloor \bmod 7 \end{aligned}$$

For example, $94 = 12\times 7 + 10$, so that doomsday 1994 is $3 + 7 + 10 + 2 \equiv 1 \bmod 7$; that is, doomsday 1994 is Monday, as we saw above.

We know doomsday of a particular year, we can use various tricks (e.g., my Uncle Charles) to pass from doomsday to any other day in the year. Conway observes that some other dates falling on the same day of the week as the doomsday are

April 4, June 6, August 8, October 10, December 12,

May 9, July 11, September 5, and November 7;

it is easier to remember these dates using the notation

4/4, 6/6, 8/8, 10/10, 12/12, and 5/9, 7/11, 9/5, and 11/7,

where m/d denotes month/day (we now return to the usual counting having January as the first month :1 = January). Since doomsday corresponds to the last day of February, we are now within a few weeks of any date in the calendar, and we can easily interpolate to find the desired day.

4. The Doomsday Algorithmas a poem

John Conway introduced the Doomsday Algorithm with the following rhyme:

(1) The last of February or of January will do
(2) (Except that in Leap Years it's January 32).
(3) Then for even months use the month's own day,
(4) And for odd ones add 4, or take it away.

(5) Now to work out your doomsday the orthodox way
(6) Three things you should add to the century day
(7) Dozens, remainder, and fours in the latter,
(8) (If you alter by sevens of course it won't matter)

(9) In Julian times, lackaday, lackaday
(10) Zero was Sunday, centuries fell back a day
(11) But Gregorian 4 hundreds are always Tues.
(12) And now centuries extra take us back twos.
(13) According to length or simply remember,
(14) you only subtract for September, or November.

4.1 The Doomsday

In other words here is a simple trick that we can use to determine the day of the week for any date of the current year. The day of the week on which the last day of February falls is called the **doomsday**. For non-leap years (or common years), this date is February 28; for leap years, it is February 29. Since 2017 is a common year, the current doomsday is the day of the week on which February 28, 2017 occurred; a Tuesday. Everything else that we need follows from one simple lemma:

4.2 Lemma

Adding or subtracting any integer multiple of 7 to any date leaves the day of the week unchanged. For example, February 7, 14, 21, and 28 all fall on the same day of the week. Likewise, adding x days is equivalent to adding $x - 7$ days, which is equivalent to subtracting $7- x$ days. For example, with $x = 6$, the day of the week that falls 6 days after Monday, is the same as the one that is $7- 6 = 1$ day before.

4.3 The rule of January

We now use the Lemma 4.2 to identify at least one date in every other month that falls on the same day of the week as the doomsday. We begin with the **rule of January**. In a common year (like 2013) February has 28 days. By applying the above lemma, we subtract 4 7 = 28 days from February 28 to arrive at February 0, which must also fall on the doomsday. But February 0 is just another name for January 31 as both dates immediately precede February 1. Thus in a common year, January 31 is the doomsday. In leap years, the doomsday is February 29. Again subtracting 28, a multiple of 7, yields February 29 – 28 = February 1. Thus in a leap year, February 1 falls on the doomsday. With a touch of whimsey, this date is also called "January 32", as both dates immediately follow January 31. Thus in leap years January 32 is the doomsday. Alternatively, observe that

$$31 - 28 = 3, \text{ and } 32 - 28 = 4.$$

Thus January 3 always falls on the same day of the week as January 31; as January 4 does for February 1. Consequently, for common years (which come in groups of 3), the doomsday is January 3; and for leap years (which come once every 4 years) the doomsday is January 4.

4.4 The rule of March

Since the doomsday immediately precedes March 1(for both leap years and non-leap years), we call the last day of February "March 0". Thus, for every year, March 0 is the doomsday. If we insist on using an actual date in March, Lemma 4.2 implies that March 7, 14, 21, and 28 any multiple of 7) are all doomsdays.

4.5 The rule of even months

The third line of Conway’s rhyme expresses the **rule of even months**. Thus for April, the fourth month, the doomsday is on 4/4. For June it is 6/6; August, 8/8; October 10/10; and December, 12/12. The answer follows from Lemma 4.2 and an interesting pattern within the seemingly irregular distribution

Jan	Feb	Mar	Apr	May	Jun	Jul	Aug	Sep	Oct	Nov	Dec
31	28 29	31	30	31	30	31	31	30	31	30	31

Conway observed that when eight of the months are paired as shown (April with May, June with July, August with September, and October with November), then together each pair contains exactly 30 + 31 =61 days. Since adding 2 to 61 produces a multiple of 7, every doomsday in June occurs two dates after the corresponding doomsdays in April. Likewise every doomsday in August occurs two dates after a doomsday in June; etc. So all we need to do is identify one doomsday in April the remainder of even months will fall like dominoes. Using Lemma 4.2 with the rule of March we learn that March 35 is a doomsday.

To convert March 35, we carry (just like in ordinary arithmetic) 31 days (the number of days in March) from the date column, and advance the month. Thus,

$$\text{March } 35=\text{April}(35-31)=\text{April } 4.$$

Likewise doomsdays fall on June 4+2=6, August 6+2=8, October 8 + 2 = 10, and December 10 + 2 = 12, demonstrating the rule of even months.

4.6 The rule of odd months

The fourth line and footnote in Conway's rhyme describe the **rule of odd months**. Adding 4to the index of every odd month having 31 days; and subtracting 4 from the index of those that have only 30 days, yield the remaining doomsdays:

$$\begin{aligned} &\text{Mar } 3+4 = \text{Mar } 7, \text{ or } 3/7,\\ &\text{May } 5+4 = \text{May } 9, \text{ or } 5/9,\\ &\text{Jul } 7+4 = \text{Jul } 11, \text{ or } 7/11,\\ &\text{Sep } 9-4 = \text{Sep } 5, \text{ or } 9/5,\\ &\text{Nov} 11-4 = \text{Nov } 7, \text{ or } 11/7, \end{aligned}$$

Note that rule of odd months is consistent with the rule of March, and that for the remaining four months, 9is paired with 5 (5/9 and 9/5), while 7 is always paired with 11 (7/11 and 11/7). We can thus use the mnemonic "working 9 to 5 at the 7-Eleven," the latter being a national convenience store chain.

The rationale for the rule of odd months follows for each odd month: From the rule of March and Lemma 4.2, March 7, is a doomsday, Thus, adding 4 to the index of March (3) yields the doomsday March 7 =3/7.
Next, we add 63 (a multiple of 7) to March 7, obtaining March 70. Carrying the months of March and April in succession, yields

March 70=April(70−31)=May(70−31−30)=May 9,or5/9.

Advancing another 63 days, yields the doomsday May 72. Again we carry two months,

May 72=June(72−31)=July(72 – 31−30)=July 11,or 7/11.

Jul67 = Aug (67 −31)=July(67 – 31−31)= Sep5,or 9/5.

Sep68 = Oct (68 −30)=July(68 – 30 −31)= Nov7,or 11/7.

See Table 1 for a summary of the doomsdays obtained for each month.

5. Finding the doomsday in a future or past year

The second stanza of Conway's poem describes how to find the doomsday for an arbitrary year. The basic fact to remember is that common years, like 2011, 2013, and 2014, have exactly 365 days. It is easy to verify that $365 = 7 \times 52 + 1$. Thus if the following year is a common year, then the doomsday advances by one day of the week. So in 2014 the last day of February, February 28, 2014, will fall on a Friday, and all of the dates shown in Table 1 will be Fridays in the year 2014. Likewise, if the current year is a common year, then the doomsday of the previous year retreats by one day of the week. Thus, the doomsday for 2012 (February 29, 2012) was a Wednesday.

Leap years on the other hand have $366 = 7 \times 52 + 2$ days. Thus if the following year is a leap year, then its doomsday will advance by two days of the week. And if the current year is a leap year then the previous year's doomsday would be two days earlier in the week. Thus, the doomsday of 2011 was Wednesday – 2 = Monday. The following table (in which leap years appear in bold typeface) illustrates this.

Doomsdays				
Jan. $\begin{Bmatrix} 3 \text{ or } 31 \\ 4 \text{ or } 32 \end{Bmatrix}$	Rule of January		Jul. 11	Rule of odd months (7+4)
Feb. $\begin{Bmatrix} 28 \\ 29 \end{Bmatrix}$	Basic definition		Aug. 8	Rule of even months (8/8)
Mar. 7	Rule of odd months (3+4)		Sept. 5	Rule of odd months (9−4)
Apr. 4	Rule of even months (4/4)		Oct. 10	Rule of even months (10/10)
May. 9	Rule of odd months (5+4)		Nov. 7	Rule of odd months (11−4)
Jun. 6	Rule of even months (6/6)		Dec. 12	Rule of even months (12/12)

Table 1: A summary of the doomsday rules applied to each month of the year. For those dates appearing in curled braces, the upper value should be used in a non-leap year, and the lower, in a leap year.

Year Doomsday		**1988** Mon.	1989 Tue.	1990 Wed.	1991 Thu.	**1992** Sat.	1993. Sun.	1994 Mon.	1995 Tue.	**1996** Thu.	1997 Fri.	1998 Sat.	1999 Sun.
Year Doomsday		**2000** Tue.	2001 Wed.	2002 Thu.	2003 Fri.	**2004** Sun.	2005 Mon.	2006 Tue.	2007 Wed.	**2008** Fri.	2009 Sat.	2010 Sun.	2011 Mon.
Year Doomsday		**2012** Wed.	2013 Thu.	2014 Fri.	2015 Sat.	**2016** Mon.	2017 Tue.	2018 Wed.	2019 Thu.	**2020** Sat.	2021 Sun.	2022 Mon.	2023 Tue.

Table 2: The day of the week on which the doomsdays listed in Table 1 fall on. Leap years are identified in bold font

6. The twelve-year rule

Also note any 12-year jump forward (up to the 99th year in a century) advances the doomsday by one day of the week for both leap or non-leap years. Actually, we don't need the table to figure this out. Every such 12year period contains exactly 3 leap years, and therefore exactly 12 – 3=9 non-leap years. So moving forward by twelve years advances the doomsday by 3×2+9×1=15 days. Subtracting two sets of 7(remember adding or subtracting 7 does not change the weekday) yields 15−14 =1. So the day advances by 1, and thus the doomsday in 2026will fall on a Saturday. Going backwards by 12 results in a 1 day retreat, so the doomsday in2014−12=2002 was Thursday. We'll call this the twelve-year rule.

7. Computing the doomsday for an arbitrary year

To simplify computing the doomsday for years in different centuries, Conway's algorithm uses the last year of each century as a reference. It is not difficult to verify that the doomsdays for these years obey the following pattern, (see lines 11 and 12 in the poem):

GREGORIAN CENTURIES BY DOOMSDAY						
SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
1700		1600	1500			
2100		2000	1900		1800	
2500		2400	2300		2200	

Note that every Gregorian century mark divisible by 400 is a leap year, and has a doomsday of Tuesday. Furthermore, the doomsday retreats by two weekdays with every advancing (non-leap year) century. The ultimate short cut is expressed in the second stanza (lines 5–8). For July 4, 1776. Start with the century mark 1600; the doomsday is Tuesday. Moving forward to 1700, the doomsday falls back two days to Sunday. Now find the largest multiple of 12 that is less than or equal to 76 (that is 1776 − 700). Clearly 76 = 12 × 6 + 4. So the doomsday advances 6 days, for the quotient, plus 4 days for the remainder, plus 1 more day because 1776 is in fact a leap year. Thus the doomsday of 1776 is Sunday plus 11 days, which by the lemma equals Sunday minus three days, or Thursday. Since July 4, is always a doomsday, July 4, 1776 was a Thursday.

Sometimes we may see the notation $\lfloor 76/12 \rfloor=6$, which means that the greatest integer contained in the quotient 76/12 =6.333...is 6. The function $\lfloor x \rfloor$is called the **floor of x**. Likewise, we frequently represent the remainderby the mod, or **modulus**. Explicitly76 mod 12 =4. Consequently, the entire doomsday calculation for July 4, 1776 can be written as

$$\text{Sunday}+\lfloor 76/12 \rfloor+76\text{mod}12+\lfloor (76\text{mod}12)/4 \rfloor=\text{Sunday}+6+4+1=\text{Sunday}+11=\text{Thursday}.$$

Finally, for the Julian calendar (which was still used in English speaking countries and colonies up to 1750), the doomsdays retreated by one day every century.

JULIAN CENTURIES BY DOOMSDAY						
SUNDAY	MONDAY	TUESDAY	WEDNESDAY	THURSDAY	FRIDAY	SATURDAY
0						
700	600	500	400	300	200	100
1400	1300	1200	1100	1000	900	800
				1700	1600	1500

8. Methods for Accelerating Conway's Doomsday Algorithm

We propose a modification of a key component in the Doomsday Algorithm for calculating the day of the week of any given date. In particular, we propose to replace the calculation of the required expression:

$$\left\lfloor \frac{x}{12} \right\rfloor + x \bmod 12 + \left\lfloor \frac{x \bmod 12}{4} \right\rfloor$$

with

$$2y + 10(y \bmod 2) + z + \left\lfloor \frac{2(y \bmod 2) + z}{4} \right\rfloor$$

where x is an input 2-digit year;

y is the tens digit of x;

z is the ones digit of x;

We argue the fact that our modification operates on individual base-10 digits makes the algorithm easier to calculate mentally.

The Doomsday algorithm's input is a calendar date of the form MM/DD/YYYY where MM is the month, DD is the day, and YYYY is the year. YYYY can further be broken down to its constituent century CC and year within the century YY. The output of the algorithm is a number between 0 to 6 that corresponds to each of the 7 days for the week.

The key equation of the Doomsday algorithm can be described a sum (modulo 7) of three terms:

day_of_the_week = (doomscentury + doomsyear + doomsmonth) mod 7

where:

doomscentury(CC) is a function of the input date's century

doomsyear(YY) is a function of the input date's 2-digit year within a century

doomsmonth(MM,DD) is a function of the input date's calendar month and day.

The doomsyear formula provided by Conway is:

$$\left\lfloor \frac{x}{12} \right\rfloor + x \bmod 12 + \left\lfloor \frac{x \bmod 12}{4} \right\rfloor$$

where x is the input date's 2-digit year within a century. The addition is always modulo 7, so the resulting sum is a number between 0 and 6, inclusive.

8.1 Simple Method

Let us break down the two-digit year x into its constituent digits y and z, where y is the tens digit, and z is ones digit. That is,

$$y = \left\lfloor \frac{x}{10} \right\rfloor$$

$$z = x \bmod 10$$

For example, if x = 74, then y= 7 and z = 4. If x = 88, then y= 8 and z = 8.

Having defined y and z in terms of x, we propose the replacement doomsyear function as

doomsyear (y,z) = 2 y + 10(y mod 2) + z + leaps

where (y mod 2) is really just a decision function to tell whether y is odd or even.

(y mod 2) = 1 if y is odd

(y mod 2) = 0 if y is even

We define an extra variable called leaps as the number of leap years between the start of the y decade and the z year. If the start of a decade is a leap year, we don't count it. But if the year z is a leap year, we do include it. For example, if x = 88, the decade starts at 80 and we have leaps = 2 because 84 and 88 are leap years. If x = 74, the decade starts at 70 and we have leaps = 1 because 72 is a leap year. In general, the variable leaps can only have three values: 0, or 1, or 2. There can't be more than 2 leap years after the start of a decade. Remember, we never include the start of the decade in our leap count.

The explicit formula for leaps is

$$\text{leaps} = \left\lfloor \frac{2(y \bmod 2) + z}{10} \right\rfloor$$

8.2 Examples.

Let's calculate the doomsyear term for these years:

1) 1974: y = 7, z = 4
   doomsyear = 2*7 + 10*1 + 4 + leaps = 14 + 10 + 4 + 1 = 29 = **1**(mod 7)
   leaps = 1 because 1972 is a leap year
2) 2040: y = 4, z= 0
   doomsyear = 2*4 + 10*0 + 0 + leaps = 8 + 0 + 0 + 0 = **1**(mod 7)
   leaps = 0
3) 2010: y = 1, z = 0
   doomsyear = 2*1 + 10*1 + 0 + leaps = 2 + 10 + 0 + 0 = 12 = **5**(mod 7)
   leaps = 0
4) 1988: y = 8, z= 8
   doomsyear = 2*8 + 10*0 + 8 + leaps = 16 + 0 + 8 + 2 = 26 = **5**(mod 7)
   leaps = 2 because 1984 and 1988 are leap years
5) 2007: y = 0, z= 7
   doomsyear = 2*0 + 10*0 + 7 + leaps = 0 + 0 + 7 + 1 = 8 = **1**(mod 7)
   leaps = 1 because 2004 is a leap year
6) 1998: y = 9, z=8
   doomsyear = 2*9 + 10*1 + 8 + leaps = 18 + 10 + 8 + 2 = 38 = **3** (mod 7)
   leaps = 2 because 1992 and 1996 are leap years.

Let's define the decade anchor to be the 2y + 10(y mod 2) subexpression of our doomsyear term. This subexpression only depends on the y decade of the input year.

$$\text{decade\_anchor}(y) = (\,2\,y + 10\,(y \bmod 2)\,) \bmod 7$$

Here is the table to memorize:

y	decade	2y + 10(y mod 2)	decade anchor
0	00's	0	0
1	10's	12	5
2	20's	4	4
3	30's	16	2
4	40's	8	1
5	50's	20	6
6	60's	12	5
7	70's	24	3
8	80's	16	2
9	90's	28	0

Table : Decade anchor lookup

If we memorize this simple table of 10 numbers, we can avoid calculating the decade anchor 2y + 10 (y mod 2) component of the doomsyear formula.

Thedoomsyeaformula is thus:

doomsyear(y,z) = decade_anchor(y) + z + leaps

Let's look at a table of possible values for the leap term depending on the digit year z:

digit year z	leaps	leaps if y is even	leaps if y is odd
0	0	0	0
1	0	0	0
2	0 or 1	0	1
3	0 or 1	0	1
4	1	1	1
5	1	1	1
6	1 or 2	1	2
7	1 or 2	1	2
8	2	2	2
9	2	2	2

Table : Possible Values for leap

year	Conway's doomsyear	2y + 10 (y mod 2) + z + leaps	year	Conway's doomsyear	2y + 10 (y mod 2) + z + leaps
0	0	0	56	0	0
1	1	1	57	1	1
2	2	2	58	2	2
3	3	3	59	3	3
4	5	5	60	5	5
5	6	6	61	6	6
6	0	0	62	0	0
7	1	1	63	1	1
8	3	3	64	3	3
9	4	4	65	4	4
10	5	5	66	5	5
11	6	6	67	6	6
12	1	1	68	1	1
13	2	2	69	2	2
14	3	3	70	3	3
15	4	4	71	4	4
16	6	6	72	6	6
17	0	0	73	0	0
18	1	1	74	1	1
19	2	2	75	2	2
20	4	4	76	4	4
21	5	5	77	5	5
22	6	6	78	6	6
23	0	0	79	0	0
24	2	2	80	2	2
25	3	3	81	3	3
26	4	4	82	4	4
27	5	5	83	5	5
28	0	0	84	0	0
29	1	1	85	1	1
30	2	2	86	2	2
31	3	3	87	3	3
32	5	5	88	5	5
33	6	6	89	6	6
34	0	0	90	0	0
35	1	1	91	1	1
36	3	3	92	3	3
37	4	4	93	4	4
38	5	5	94	5	5
39	6	6	95	6	6
40	1	1	96	1	1
41	2	2	97	2	2
42	3	3	98	3	3
43	4	4	99	4	4
44	6	6			
45	0	0			
46	1	1			
47	2	2			
48	4	4			
49	5	5			
50	6	6			
51	0	0			
52	2	2			
53	3	3			
54	4	4			
55	5	5			

Table 3: Doomsyear Values from 00 to 99

8.3 Conway's Look-up Table Acceleration Method

John Horton Conway devised an acceleration method to speed-up the calculation of the doomsyear term. In practice, Conway's method is probably the fastest acceleration method for such, but it involves memorizing 18 numbers and some non-intuitive rules. Now, we will describe Conway's look-up table method. And then, we will compare and contrast our method with Conway's acceleration method.

Conway's lookup table method requires memorizing the years of the century where the doomsyear value is zero. Let us call these numbers as zero-anchor years. These are:

0	6	11.5	17	23	28	34	39.5	45
51	56	62	67.5	73	79	84	90	95.5

There's actually the added complication of the half-numbers: 11.5, 39.5, 67.5 and 95.5. These half-numbers mean that the preceding year has doomsyear value 6 and the succeeding year has doomsyear value 1. For example, doomsyear (11) = 6, and doomsyear(12) = 1; doomsyear (67) = 6, and doomsyear (68) = 1. These half-numbers occur because of the increment-by-2 property of doomsyear values during leap years. In effect, a doomsyear of value 0 got skipped in the half-number locations.

Here are the steps of Conway's acceleration method:

1) Select the nearest zero-anchor year less than your input year.

2) Let z_0 be the difference between your input year and the selected zero-anchor. Ignore fractional values of half-numbers in zero-anchor years. That is, treat 11.5, 39.5, 67.5, and 95.5 as 11, 39, 67, and 95 respectively in calculating the difference

3) Count the number of leap years between the zero-anchor and your input year. If the zero-anchor year is a leap year, do not include it. On the other hand, if your input year is a leap year, include it in the leap count. Let us denote this count of leap years as $leap_0$

4) Add z_0 and $leap_0$ to get the doomsyear value. If the selected zero-anchor is a half- number, subtract 1 from the sum. We denote this as the anchor adjustment term needed for half-number zero-anchors.

To sum it all up, Conway's acceleration method can be described by the equation:

$$doomsyear = anchor\_adjustment + z_0 + leap_0$$

Note that the anchor_adjustment term is almost always zero except for half-number years where it has a value of –1.

8.4 Examples

1) 1974:
   zero-anchor is 1973
   $z_0 = 1974 - 1973 = 1$
   $leap_0 = 0$
   doomsyear = 1 + 0 = **1**

2) 2040:
   zero-anchor is 2039.5
   $z_0 = 2040 - 2039 = 1$
   $leap_0 = 1$ because 2040 is a leap year
   doomsyear = – 1 + 1 + 1 = **1**

3) 2010:
   zero-anchor is 2006
   $z_0 = 2010 - 2006 = 4$
   $leap_0 = 1$ because 2008 is a leap year
   doomsyear = 4 + 1 = **5**

4) 1988:
   zero-anchor is 1984
   $z_0 = 1988 - 1984 = 4$
   $leap_0 = 1$ because 1988 is a leap year. Remember, we don't count the zero-anchor
   doomsyear = 4 + 1 = **5**

5) 2007:
zero-anchor is 2006
$z_0 = 2007 - 2006 = 1$
$leap_0 = 0$
doomsyear = 1 + 0 = **1**

6) 1998:
zero-anchor is 1995.5
$z_0 = 1998 - 1995 = 3$
$leap_0 = 1$ because 1996 is a leap year
doomsyear = – 1 + 3 + 1 = **3**

7) 1914:
zero-anchor is 1911.5
$z_0 = 1914 - 1911 = 3$
$leap_0 = 1$ because 1912 is a leap year
doomsyear = – 1 + 3 + 1 = **3**

8) 1972:
zero-anchor is 1967.5
$z_0 = 1972 - 1967 = 5$
$leap_0 = 2$ because 1968 and 1972 are leap years
doomsyear = – 1 + 5 + 2 = **6**

9. A Comparison of Simple Acceleration Method with Conway's

Simple method is amenable to lookup table acceleration. In fact, we claim that after this lookup table acceleration, simple method is very similar in form to Conway's acceleration method. Let us compare and contrast the doomsyear equation for simple method and Conway's method. These are:

$$\text{doomsyear}(y,z) = \text{decade\_anchor}(y) + z + \text{leaps}$$

and

$$\text{doomsyear}(x) = \text{anchor\_adjustment}(x) + z_0 + leap_0$$

respectively.. Let us list down these similarities.

1) Both equations are the sum of 3 terms. Each of these terms corresponds with counterpart in the other method.

2) Both equations use memorization of an anchor year for the speedup. In simple method, the starting year of the decade serves as the anchor. In Conway's method, years with doomsyear value of zero are used as the anchor.

3) Both equations use z as the number of years between the input year and the anchor year. We can consider z as the offset from the anchor.

4) Both equations contain a leap count correction term that counts the number of leap years between the anchor year and the input year.

We now list down differences between the 2 equations and mention some advantages of our method over Conway's method.

1) In simple method, z is not computed. It is part of the input. In Conway's method, z_0 has to be calculated by subtracting the zero-anchor year from the input year.

2) In simple method, one has to memorize 10 digits for the anchoring. In Conway's method, one has to memorize 18 numbers for the anchoring.

3) In simple method, the leap count correction term follows a regular pattern for a given decade and is amenable to another speedup via memorization.

4) In simple method, we can always fall back to using the 2y + 10 (y mod 2) calculation if entries in the lookup table are forgotten.

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